

# Decoherence at absolute zero

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## Abstract

We present an analytical study of the loss of quantum coherence at absolute zero. Our model consists of a harmonic oscillator coupled to an environment of harmonic oscillators at absolute zero. We find that for an Ohmic bath, the offdiagonal elements of the density matrix in the position representation decay as a power law in time at late times. This slow loss of coherence in the quantum domain is *qualitatively* different from the exponential decay observed in studies of high temperature environments.

## I. INTRODUCTION

Quantum mechanics gives an excellent description of the microscopic world. In all experimental situations studied so far, the predictions of quantum theory have been verified with phenomenal accuracy. However, ever since the theory came into being it has caused a deep unease among physicists. This dissatisfaction originates from a belief that the foundations of the theory are infirm, or at best poorly understood. In order to verify the predictions of quantum mechanics, one has to perform *measurements*. However, quantum mechanics does *not* provide a description of the measuring process itself. Apart from a dynamical descrip-

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tion of the quantum system via Schrödinger's equation, one needs to postulate [1] *ad hoc* that the state of the system “collapses” into one of the possible classical outcomes selected by a measuring apparatus. This point of view, which is popularly known as the Copenhagen interpretation, explicitly requires the existence of a classical measuring apparatus. It is clearly unsatisfactory that for its interpretation quantum mechanics needs to rest upon classical mechanics, which it is supposed to supersede. A more general theory usually subsumes a less general one as for instance, the special theory of relativity contains Newtonian physics as a special case. One can formulate the special theory of relativity independent of Newtonian physics. The situation is much less satisfactory for quantum mechanics. The basic problem stems from the superposition principle, which is one of the corner stones of quantum mechanics. This principle states that if two states of a system are allowed, an arbitrary linear combination of them also is. As a consequence, these two allowed states can occur with a definite phase relation. It is this “coherent superposition” of alternatives which is in apparent conflict with everyday experience. Coherent superposition gives rise to the possibility of interference between alternatives, in some sense that both alternatives simultaneously occur. This is best illustrated by Schrödinger's cat paradox. One can devise a situation in which the life of a cat depends on whether a radioactive nucleus has decayed. This makes the state of a macroscopic object (the cat) depend on the state of a microscopic one (the nucleus). If the state of the nucleus is in a superposition of two states (decayed and not decayed), the cat too will have to be in a superposition of two states - alive and dead. This thought experiment brings out the absurd nature of quantum mechanics, when it is applied to the macroscopic world. This is sometimes referred to in the physics community as “the measurement problem” [2]. There have been many attempts to address this problem, but unfortunately, with very little progress.

In the last few decades, a fresh perspective has emerged on this issue [3]. This new point of view uses quantum Brownian motion as a model for understanding the loss of quantum coherence due to a system's contact with its environment. This interest has been triggered

by the possibility of probing the boundary between the microscopic and the macroscopic worlds via increasingly sophisticated experiments [4]. In this open-system approach to quantum measurement, one models the quantum system as a free particle in a potential  $V(x)$  and its environment as a collection of harmonic oscillators [5,6]. The effective dynamics of the quantum system is then studied by “ignoring” the environmental degrees of freedom. More precisely, the dynamics of the reduced density matrix (which is obtained from the full density matrix of the system plus the environment by tracing out the environmental degrees of freedom) is analyzed. This approach leads to the conclusion that quantum coherence between two Gaussian wave-packets separated by a distance  $\Delta x$ , decays exponentially over a time-scale known as the decoherence time  $\tau_D = \gamma^{-1}(\lambda_{\text{Th}}/\Delta x)^2$ , where  $\lambda_{\text{Th}}$  is the thermal de Broglie wavelength and  $\gamma$  the relaxation rate of the system. In other words, a superposition of states cannot be observed if the system is probed at time scales greater than  $\tau_D$ . Thus one seems to get an explanation for the loss of quantum coherence *within* the framework of quantum mechanics. However, most of these studies take the high temperature ( $T \rightarrow \infty$ ) limit for analytic convenience. While such an assumption leads to some technical simplifications it is inadequate from a conceptual point of view. Since the high temperature limit ( $T \rightarrow \infty$ ) is equivalent to the classical limit ( $\hbar \rightarrow 0$ ) such an analysis seems to require classical mechanics for understanding the decoherence aspect of quantum mechanics just as in the Copenhagen interpretation.

In this paper we address and partially answer the following questions: Is there a loss of quantum coherence even at absolute zero? If so, what is the law of decoherence in the absence of thermal fluctuations? The main motivation for this study is the following. Since an environment at absolute zero is at the extreme quantum limit (where the analysis simplifies), the present study enables us to understand the process of loss of quantum coherence entirely *within* the framework of quantum mechanics. This is a conceptually important point which sets the present work apart from most earlier attempts towards understanding the measurement aspect of quantum mechanics via environment induced decoherence. Unruh

and Zurek [7], Paz, Habib and Zurek [8] and Caldeira and Leggett [9] have addressed similar questions in the past. However, the work of Unruh and Zurek [7] is based on an incorrect regularization procedure which is in conflict with the fluctuation-dissipation theorem. At the end of this paper we discuss how the work discussed here relates to Refs. [8,9].

We consider a harmonic oscillator coupled to an environment characterized by an Ohmic spectral density (i.e  $I(\omega) = (2/\pi)\Gamma\omega\theta(\Lambda - \omega)$ , with  $\Gamma$  the friction coefficient and  $\Lambda$  the upper cut-off on the frequency spectrum of the environmental oscillators). The environment is coupled to the system via a coordinate- coordinate coupling. In the limit of weak coupling between the system and the environment, we find that the quantum system exhibits a new behavior for the loss of quantum coherence at absolute zero.

## II. EVOLUTION OF THE REDUCED DENSITY MATRIX

There have been several attempts at deriving a general master equation for a quantum system coupled to a harmonic oscillator environment [5-7,10]. One of the first attempts in this direction was due to Caldeira and Leggett [6] who gave an explicit derivation of the evolution of the reduced density matrix valid for a high-temperature environment. Since their work, there have been many derivations under more and more general conditions [7,10]. Unfortunately, many of these derivations are internally inconsistent. As mentioned earlier, the derivation of Unruh and Zurek [7], which is supposed to be valid for an Ohmic environment at all temperatures makes use of an incorrect regularization procedure which violates the fluctuation-dissipation theorem. The most general and correct derivation so far has been due to Hu, Paz and Zhang (HPZ) [10]. HPZ have incorporated all the subtleties which come into play at low temperatures. These subtleties stem from the *nonlocality* of the noise kernel at low temperatures.

In this paper we base our analysis on the master equation derived by HPZ. The Hamiltonian for the composite system described by HPZ is given by

$$H = H_S + H_I + H_E, \quad (1)$$

with  $H_S$  and  $H_E$  representing the Hamiltonian of the system and the environment respectively.  $H_I$  is the Hamiltonian which pertains to interaction between the system and the environment. The explicit expressions for  $H_S$ ,  $H_I$  and  $H_E$  are

$$H_S = \frac{P^2}{2M} + \frac{M\Omega^2 x^2}{2}, \quad H_I = \sum_n C_n x q_n, \\ H_E = \sum_n \left( \frac{p_n^2}{2m_n} + \frac{m_n \omega_n^2 q_n^2}{2} \right).$$

Here  $M$ ,  $x$  and  $P$  are respectively the mass, position and the momentum of the system while  $m_n$ ,  $q_n$  and  $P_n$  are respectively the mass, position and momentum of the  $n$ th environmental oscillator.  $\Omega$  and  $\omega_n$  are the frequency of the system oscillator and the  $n$ th environmental oscillator respectively.  $C_n$  is the coupling constant between the system and the  $n$ th environmental oscillator. The system and the environment are assumed to be decoupled initially. At  $t = 0$  interaction is turned on. The corresponding evolution equation for the reduced density matrix for the quantum system in the coordinate representation is

$$i\hbar \frac{\partial \tilde{\rho}(x, x', t)}{\partial t} = \left[ -\frac{\hbar^2}{2M} \left( \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial x'^2} \right) \right. \\ \left. + \frac{M}{2} \Omega^2 (x^2 - x'^2) \right] \tilde{\rho}(x, x', t) \\ + \frac{M}{2} \delta\Omega(t)^2 (x^2 - x'^2) \tilde{\rho}(x, x', t) \\ - i\hbar\Gamma(t)(x - x') \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right) r\tilde{h}o(x, x', t) \\ - iD(t)(x - x')^2 \tilde{\rho}(x, x', t) \\ + \hbar A(t)(x - x') \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial x'} \right) \tilde{\rho}(x, x', t). \quad (2)$$

The terms in the square brackets make up the Liouvillian. The second term is a time dependent frequency shift of the system oscillator as a result of its coupling with the environment. The third term represents dissipation with a time dependent dissipative coefficient. The fourth term describes diffusion. The last term in the equation is named “anomalous diffusion” in the literature since it generates a second derivative term in the phase space representation of the evolution equation just like the ordinary diffusion term. The time dependence of the coefficients entering this equation is in general rather complicated [10]. However, one can get simpler functional forms for these coefficients in some limiting cases. We will write down the explicit forms of the coefficients relevant to the present problem later in this paper.

The evolution of the system involves the following time scales: the scale associated with the natural frequency  $\Omega$  of the system oscillator, the relaxation time scale  $\gamma^{-1}$  which is controlled by the coupling strength between the system and the environment, a “memory time”, which is the inverse of the highest frequency  $\Lambda$  of the reservoir and the thermal time scale  $\tau_\beta = \hbar\beta$  (with  $\beta = 1/k_B T$  where  $k_B$  is the Boltzmann’s constant and  $T$  the temperature of the bath). In this paper we focus on the time regime  $t \gg \tau$  where  $\tau = \gamma^{-1}$  is the typical response time of the system. In other words, we are interested in the dynamics of loss of coherence at long times after some initial transients have died out.

We now study how an initial coherent superposition of two wave-packets evolves into an incoherent mixture at an arbitrary temperature. In order to make analytical progress we need to make some reasonable approximations. For large enough separation (more precisely for  $|(x - x')| \gg \lambda_q$  where  $\lambda_q = \sqrt{\hbar/M\gamma}$ )<sup>1</sup> between the constituent wavepackets one can

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<sup>1</sup>In the high temperature limit, one finds that coherence between two wavepackets is lost if they are separated by a distance greater than the thermal de Broglie wavelength ( $\hbar/\sqrt{2Mk_B T}$ ). A naive

approximate Eq. (2) as follows,

$$i\hbar \frac{\partial \tilde{\rho}(x, x', t)}{\partial t} \simeq -D(t)(x - x')^2 \tilde{\rho}(x, x', t). \quad (3)$$

This leads to the conclusion that the off-diagonal elements of the density matrix in the position representation get suppressed according to

$$\begin{aligned} \tilde{\rho}(x, x', t) &= \tilde{\rho}(x, x', 0) \\ &\times \exp \left( -\frac{(x - x')^2}{\hbar} \int_0^t D(t') dt' \right). \end{aligned} \quad (4)$$

In the high temperature regime, Eq. (4) leads to the usual exponential damping of the off-diagonal elements of the density matrix [3]. In the limit of weak coupling between the system and the environment (more precisely, if one works up to the second order in the coupling constant between the system and the environment) one gets [10]

$$D(t) = \int_0^t \nu(s) \cos \Omega s ds, \quad (5)$$

where  $\nu(s)$  is the noise kernel given by

$$\nu(s) = \int_0^\infty I(\omega) \coth \frac{\beta \hbar \omega}{2} \cos \omega s d\omega. \quad (6)$$

Furthermore we restrict ourselves to the case of an Ohmic environment characterized by a spectral density  $I(\omega) = (2/\pi)\Gamma\omega\theta(\Lambda - \omega)$ , with  $\Gamma$  the friction coefficient and  $\Lambda$  the upper cut-off on the frequency spectrum of the environmental oscillators. Such an environment leads to the classical law of friction in the limit of high temperatures.

Thus for an Ohmic environment Eq. (6) goes over to

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extension of this formula to the low temperature regime would imply that the coherence length goes to infinity, i.e. there is no destruction of coherence at *any* separation. Our analysis of the extreme quantum domain shows that at zero temperature the coherence length is *finite* and given by  $\sqrt{\hbar/M\gamma}$ .

$$\nu(s) = (2/\pi)M\gamma \int_0^\Lambda \omega \coth \frac{\beta\hbar\omega}{2} \cos \omega s d\omega, \quad (7)$$

where the dissipative coefficient  $\gamma = \Gamma/M$ .

The explicit form of the integral appearing in the argument of the exponential in Eq. (4) at absolute zero is

$$(2/\pi)M\gamma \int_0^t \int_0^{t'} \int_0^\Lambda \omega \cos \omega s \cos \Omega s ds d\omega. \quad (8)$$

If we confine ourselves to the regime  $t$  large compared to the relaxation time  $\tau$  and the memory time  $\Lambda^{-1}$  but still small compared to  $\Omega^{-1}$ , where  $\Omega$  is the infrared cutoff<sup>2</sup>, Eq. (8) reduces to

$$(2/\pi)M\gamma \int_0^t \frac{1 - \cos \Lambda t'}{t'} dt'. \quad (9)$$

We find that the contribution from the rapidly oscillating cosine term is negligible compared to that from the first term in (9) as  $\Lambda \rightarrow \infty$ . We thus arrive at the following time dependence for the off-diagonal elements of the reduced density matrix at late times (i.e.  $\Omega^{-1} > t \gg \tau > \Lambda^{-1}$ ),

$$\tilde{\rho}(x, x', t) \sim t^{-\alpha}, \quad (10)$$

where  $\alpha = (2/\pi\hbar)M\gamma(x - x')^2$ .

This is the central result of the paper. We notice that, in contrast to the high temperature case, there is *no single characteristic decoherence time scale at absolute zero*. The slow power-law loss of quantum coherence at absolute zero is *qualitatively* different from the rapid suppression of coherence that one sees at high temperatures. This extremely slow decoherence stems from the *finite memory* in the system at absolute zero. More precisely, it can

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<sup>2</sup>In this time regime the system essentially behaves like a free particle as it does not “see” the confining potential.



be traced to the temporal nonlocality of the noise kernel. The exponent  $\alpha$  appearing in the decay of the off-diagonal elements of the density matrix is independent of the temperature reflecting the essential *quantum* nature of the dynamics.

Let us discuss some of the consequences of this result. First of all it is intriguing that even at absolute zero a coherent superposition of states can get destroyed by zero point fluctuations. In other words, one is led to an understanding of the transition of an initial pure state into an incoherent mixture of states *within* the realm of quantum theory. Secondly, since this loss of coherence is very slow, we find that the effect of quantum zero point fluctuations on decoherence is considerably weaker than that of thermal fluctuations. There is one important difference between decoherence induced by a high temperature environment and that by a low temperature one. At absolute zero the quantum system can only lose energy to the cold environment. In contrast at higher temperatures the environment can also induce transitions between energy levels available to the quantum system. We would like to emphasize that a zero temperature environment is an extreme quantum system. Such an environment is ideal for an entirely quantum mechanical analytical approach towards the decoherence process. This fact has not been adequately appreciated in those earlier approaches towards the decoherence problem which are based on a high temperature (i.e. classical) limit for the environment. In the short time regime ( $\gamma t \ll 1$ ) studied by Caldeira and Leggett [9] one finds that dephasing occurs over a time scale of the order of the time for spontaneous emission of one quantum of energy from the system to the environment at absolute zero. On the other hand, at long times one can have spontaneous emission from a range of excited states of the system and consequently the decoherence process will be controlled by a wide range of time scales instead of a single time scale. This is perhaps the origin of the power law behavior of the decoherence process at large time scales at absolute zero. It would be interesting to find out if this slow loss of coherence is observed in experiments performed at cryogenic temperatures [4].

There are several open questions which are of interest. An analytical study of the intermediate temperature regime would enable us to probe the competition between quantum and thermal fluctuations. In the present paper we have restricted our attention only to the case of an Ohmic environment. A similar derivation with a super-Ohmic or a sub-Ohmic environment may give us different dynamics for the suppression of the off-diagonal elements of the density matrix.

In order to keep our analysis as simple as possible I we have confined ourselves to an approximate master equation (Eq. (3) ). In particular, here we have worked in a domain where the dynamics of the density matrix is dominated by diffusion. In general, however, there is a possibility of a rich interplay between the various factors which affect the dynamics of the density matrix. Therefore it would be worthwhile to do an exact analysis of the master equation at absolute zero.

In Ref. [9], Caldeira and Leggett investigate the destruction of coherence between the energy states of an oscillator due to its contact with a low temperature environment, in the weak coupling regime. They probe a time regime ( $\gamma t \ll 1$ ) which is complementary to the time regime ( $\gamma t \gg 1$ ) analyzed in the present paper. Their study reveals an exponential damping of coherence with time. After this initial exponential damping, which lasts for a time of the order of  $\gamma^{-1}$ , there *still* remains coherence of order unity ( $O(1)$ ) in the system. Then there is an intermediate time regime  $\gamma t \sim 1$  which is not analytically tractable. The present analytical study focuses on the late time regime ( $\gamma t \gg 1$ ) where we find a slow, power law decay of coherence. This suggests that in the intermediate regime the damping of coherence slows down before finally settling down at late times to a power law.

In Ref. [8], Paz, Habib and Zurek present a detailed analysis of a model of a harmonic oscillator linearly coupled to an oscillator bath. The results of their study are based on numerical calculations. One of the principal aims of Ref. [8] is to test the validity of the *high*

*temperature* approximation in the low temperature regime. They also study the spectral density dependence of the decoherence process. This work shows numerically that there *is* decoherence at absolute zero. The present study goes beyond Refs. [8,9] in presenting an analytical expression for the temporal decay of coherence at late times.

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